2. É. P. Volchkov, S. Yu. Spotar', and V. I. Terekhov, "Turbulence characteristics of a bounded twisted jet," in: Wall Jets [in Russian], Novosibirsk (1984).
3. É. P. Volchkov, Wall Gas Screens [in Russian], Novosibirsk, Nauka (1983).
4. Z. B. Sakipov, Theory and Methods of Calculation of Semi-Infinite Jets and Condensed Jets [in Russian], Nauka, Alma-Ata (1978).
5. É. P. Volchkov, N. A. Dvornikov, and V. I. Terekhov, Heat and Mass Transfer and Friction in the Turbulent Boundary Layer of a Twisted Flow, Preprint ITF SO AN SSSR, Novosibirsk (1983), No. 107-83.
6. T. Karman, "Über laminare und turbulente Reilung," Zh. Angew. Math. Mech., 1, 233 (1921).
7. D. Wormley, "Analytical model of incompressible flow in short swirl chambers," Teor. Osn. Inzh. Raschetov, No. 2 (1969).
8. G. N. Abramovich, T. A. Girshovich, S. Yu. Krasheninnikov, et al. (eds.), Theory of Turbulent Jets [in Russian], Nauka, Moscow (1984).
9. N. A. Dvornikov and V. I. Terekhov, "Transfer of momentum and heat in a turbulent boundary layer on a curved surface," Zh. Prikl. Mekh. Tekh. Fiz., No. 3 (1984).
10. E. P. Volchkov, S. S. Kutateladze, and A. I. Leont'ev, "Interaction of a turbulent jet with a hard wall," Zh. Prikl. Mekh. Tekh. Fiz., No. 2 (1965).
11. I. I. Ibragimov and B. P. Ustimenko, "Study of the aerodynamics of a twisted jet developing along a cylindrical wall in a co-current flow," Probl. Teploenerg. Prikl. Teplofiz., No. 2 (1965).
12. N. A. Dvornikov and S. Yu. Spotar', "Heat and mass transfer in a twisted wall jet," in: Hydrogasdynamics and Heat Transfer in Condensed Media [in Russian], ITF SO AN SSSR, Novosibirsk (1981).
13. N. A. Dvornikov, "Turbulent heat transfer in a twisted wall jet," in: Current Problems of Thermophysics [in Russian], ITF SO AN SSSR, Novosibirsk (1984).
14. R. A. Seban and L. M. Back, "Velocity and temperature profiles in a wall jet," Int. J. Heat Mass Transfer, 3, No. 4 (1961).
15. V. Kruka and S. Eskinazi, "The wall jet in a moving stream," J. Fluid Mech., 20, Pt. 4 (1964).

RADIAL OSCILLATIONS OF VAPOR–GAS BUBBLES

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We are concerned with vapor–gas bubbles executing small radial oscillations in a liquid. The dynamics of vapor–gas bubbles has important bearing, in particular, on sound propagation in the top layer of the ocean. The description of the process in this situation is far more complicated than in the case of a gas bubble or a vapor bubble. The attenuation of sound in a liquid containing vapor–gas bubbles is clearly related to the decay rate of the radial pulsations of the bubbles.

Here we investigate the influence of interdiffusion of the components of a vapor–gas mixture on the decay rate of small oscillations of vapor–gas bubbles. We show that the addition of a minute quantity of an inert gas to a vapor bubble lowers the damping of the bubble oscillations significantly. We confirm the fact that the derived analytical relations are in good agreement with the experimental data on the damping of radial oscillations of gas and steam bubbles in water. We also discuss the linear radial pulsations of vapor–gas bubbles in a sound field. We derive asymptotic expressions for the bubble response function, which are valid for different frequency ranges. We compare these relations with the experimental data for steam–air bubbles in subcooled water and establish good agreement between them.

1. Fundamental Equations

The problem of spherically symmetric process around vapor–gas bubbles has been formulated previously [1, 2], and their small oscillations have been investigated in detail [3-5]. The system of equations describing linear radially symmetric oscillations of a bubble filled

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with a liquid vapor and a gas that is insoluble in the liquid is given in [3], where it is assumed that a uniform pressure exists in the bubble and interdiffusion of the components of the vapor–gas mixture is taken into account.

The equations for the heat input, continuity, and the state of the phases in spherical Eulerian coordinates (r, t) have the form

$$\rho_V \frac{du_V}{dt} + \rho_g \frac{du_g}{dt} = \frac{p}{\rho}\frac{d\rho}{dt} + \frac{1}{r^2}\frac{\partial}{\partial r}\left(\lambda r^2 \frac{\partial T}{\partial r}\right) + \rho D \frac{\partial k}{\partial r}\frac{\partial}{\partial r}(u_V - u_g),$$
$$\frac{\partial \rho_V}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}\left[\rho_V (v + w_V) r^2\right] = 0, \quad 0 \leqslant r < R(t),$$
$$\frac{\partial \rho_g}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}\left[\rho_g (v + w_g) r^2\right] = 0, \quad \rho_g w_g = -\rho_V w_V = \rho D \frac{\partial k}{\partial r}, \tag{1.1}$$
$$p = p_V + p_g = (\rho_g B_g + \rho_V B_V)\, T = \rho B T,$$
$$T_V = T_g = T, \ u_V = c_{VV} T, \ u_g = c_{Vg} T, \ u_l = c_l T_l, \ \rho_l = \text{const},$$
$$\rho_l \left(\frac{\partial u_l}{\partial t} + \frac{R^2}{r^2} v_{lR} \frac{\partial u_l}{\partial r}\right) = \frac{1}{r^2}\frac{\partial}{\partial r}\left(\lambda_l r^2 \frac{\partial T_l}{\partial r}\right), \quad R < r < \infty,$$

where u is the specific internal energy, T is the temperature, v is the velocity, R is the bubble radius, D is the interdiffusion coefficient, c_p and c_V are the specific heats of the mixture at constant pressure and at constant volume, c_ℓ is the specific heat of the liquid, λ is the thermal conductivity, w is the diffusion rate, $v_{\ell R}$ is the particle velocity of the liquid on the bubble surface, B is the gas constant, p is the pressure, ρ is the density, and k is the concentration of the vapor component. The subscripts V, g, ℓ, and R refer to the parameters of the vapor, the gas, the liquid, and the phase interface, respectively.

The boundary conditions on the phase interface r = R(t) have the form in the quasi-equilibrium approximation

$$T_l = T_V = T_S(p_{VR}), \ \lambda_l \partial T_l/\partial r - \lambda \partial T/\partial r = jl, \tag{1.2}$$
$$\rho_V(\dot{R} - v - w_V) = \rho_l(\dot{R} - v_l) = j, \ \rho_g(\dot{R} - v - w_g) = 0$$

(ℓ is the specific heat of vaporization). In addition,

$$r = 0\colon \partial k/\partial r = \partial T/\partial r = 0; \ r = \infty\colon T_l = T_0. \tag{1.3}$$

The Clausius–Clapeyron equation far from any critical state, when $\rho_V \ll \rho_\ell$, can be written

$$dp_V/dT = l\rho_V/T. \tag{1.4}$$

The equation for small radial oscillations of a bubble in an incompressible liquid has the form

$$R\dot{v}_{lR} = (p - p_\infty - 2\sigma/R)/\rho_l. \tag{1.5}$$

Here p_∞ is the pressure far from the bubble, and σ is the coefficient of surface tension of the liquid.

If the homobaricity condition holds, an integral exists for the heat-input equation for the gaseous phase, which has the form for linear problems

$$\frac{dp}{dt} = \frac{3}{R}\left[-\Gamma p v_R + \frac{p(B_V - B_g)}{B} D \frac{\partial k}{\partial r}\Big|_R + (\Gamma - 1)\lambda \frac{\partial T}{\partial r}\Big|_R\right] \tag{1.6}$$

($\Gamma = c_p/c_V$ is the adiabatic exponent of the vapor–gas mixture).

In the case of small oscillations, the bubble radius can be described by the real part of the expression

$$R = R_0[1 + \delta \exp(ht)], \tag{1.7}$$

where δ is a complex number corresponding to the condition $|\delta| \ll 1$; $\omega = \mathrm{Im}\{h\}$ is the oscillation frequency, and the subscript 0 refers to the parameters in the unperturbed state.

We linearize the system (1.1)-(1.6). Let P^0, Θ^0, K^0 be small deviations of the pressure, temperature, and concentration from the equilibrium state:

$$p = p_0[1 + P^0(\tau)], \ T = T_0[1 + \Theta^0(\xi, \tau)], \ k = k_0[1 + K^0(\xi, \tau)]. \tag{1.8}$$

We assume that

$$P^0 = P\exp(H\tau),\ \Theta^0 = \Theta(\xi)\exp(H\tau),$$
$$K^0 = K(\xi)\exp(H\tau)\ (\xi = r/R_0,\ \tau = tD/R_0^2,\ H = hR_0^2/D). \tag{1.9}$$

After linearization of the system (1.1)-(1.6) and transformation to dimensionless variables, it can be rewritten as follows with allowance for relations (1.7)-(1.9) and the condition $\rho_V \ll \rho_\ell$:

$$V_R = \delta H - J/3,\ V_{lR} = \delta H; \tag{1.10}$$
$$P = HV_{lR}\Pi^{-2} - S\delta; \tag{1.11}$$
$$H\Theta = \mathrm{Le}_0\nabla^2\Theta + \left(1 - \frac{1}{\Gamma}\right)HP - k_0k_3HK; \tag{1.12}$$
$$HP = 3\Gamma\left[\mathrm{Le}_0\frac{\partial\Theta}{\partial\xi}\Big|_1 - V_R + k_0k_2\frac{\partial K}{\partial\xi}\Big|_1\right]; \tag{1.13}$$
$$HK = \nabla^2 K,\quad \nabla^2 = \frac{\partial^2}{\partial\xi^2} + \frac{2}{\xi}\frac{\partial}{\partial\xi}; \tag{1.14}$$
$$H\Theta_l = \mathrm{Le}_l\nabla^2\Theta_l; \tag{1.15}$$
$$\frac{\partial\Theta_l}{\partial\xi}\Big|_1 - \frac{\lambda}{\lambda_l}\frac{\partial\Theta}{\partial\xi}\Big|_1 = J\frac{\rho_0}{3\rho_l\,\mathrm{Le}_l k_1}; \tag{1.16}$$
$$\frac{\partial K}{\partial\xi}\Big|_1 = J\frac{(1-k_0)}{3k_0}; \tag{1.17}$$

$$\Theta_R = M\left(P + \frac{B_g}{B_0}K|_{\xi=1}\right),$$
$$k_1 = \frac{c_lT_0}{l},\quad k_2 = \frac{B_V - B_g}{B_0\Gamma},\quad k_3 = \frac{B_V - B_g}{c_p},\quad \varkappa = \frac{c_pT_0}{l},$$
$$\mathrm{Le}_0 = \frac{a_0}{D},\quad \mathrm{Le}_l = \frac{a_l}{D},\quad a_l = \frac{\lambda_l}{\rho_lc_l},\quad a_0 = \frac{\lambda_0}{\rho_0c_p},\quad S = \frac{2\sigma}{R_0p_0}, \tag{1.18}$$
$$M = \frac{B_VT_0}{l},\quad \Pi = \frac{R_0}{D}\sqrt{\frac{p_0}{\rho_l}},\quad p_0 = p_\infty + \frac{2\sigma}{R_0},$$
$$v = V\frac{D}{R_0}\exp(H\tau),\quad j = J\frac{\rho_0D}{3R_0}\exp(H\tau).$$

The solutions of Eqs. (1.12), (1.14), and (1.15) subject to the boundary conditions at $r = R_0$ and $r \to \infty$ and also the condition of finite temperature and finite concentration at the center of the bubble can be written in the form

$$\Theta = A\frac{\mathrm{sh}(\xi\sqrt{H_2})}{\xi} + \left(1 - \frac{1}{\Gamma}\right)P + \frac{k_0k_3K|_{\xi=1}}{(1-\mathrm{Le}_0)\,\xi}\left[\frac{\mathrm{sh}(\xi\sqrt{H_2})}{\mathrm{sh}\sqrt{H_2}} - \frac{\mathrm{sh}(\xi\sqrt{H})}{\mathrm{sh}\sqrt{H}}\right],$$
$$\Theta_l = \Theta_R\frac{\exp\left[\sqrt{H_1}(1-\xi)\right]}{\xi},\quad K = A_1J\frac{\mathrm{sh}(\xi\sqrt{H})}{\xi},$$
$$A = \frac{\Theta_R - (1 - 1/\Gamma)P}{\mathrm{sh}\sqrt{H_2}},\quad \Theta_R = A_3P,\quad J = A_2P,$$
$$A_1 = \frac{1-k_0}{3k_0B_1\,\mathrm{sh}\sqrt{H}},\quad H_1 = \frac{H}{\mathrm{Le}_l},\quad H_2 = \frac{H}{\mathrm{Le}_0},$$
$$A_2 = \frac{(1+\sqrt{H_1})M\lambda_l/\lambda_0 + B_2(M - 1 + 1/\Gamma)}{k_3(1-k_0)(1 - B_2/B_1)/3(1-\mathrm{Le}_0) - A_4},$$
$$A_3 = M + k_4A_2,\quad A_4 = \frac{1}{3\varkappa\mathrm{Le}_0} + k_4\left[(1+\sqrt{H_1})\frac{\lambda_l}{\lambda_0} + B_2\right],$$
$$k_4 = \frac{(1-k_0)MB_g}{3k_0B_0B_1},\quad B_1 = \sqrt{H}\,\mathrm{cth}\sqrt{H} - 1,\quad B_2 = \sqrt{H_2}\,\mathrm{cth}\sqrt{H_2} - 1.$$

The condition for the existence of a nontrivial solution of the system of linear equations yields a transcendental characteristic equation in H:

$$H + 3\Gamma\Pi^2H^{-1} - \Phi + S\Pi^2(\Phi - H)H^{-2} = 0,$$
$$\Phi = 3\mathrm{Le}_0B_2[\Gamma(M-1) + 1] + \Gamma[1 + k_2(1-k_0) +$$

$$+\mathrm{Le}_0 k_3(1-k_0)(B_2/B_1-1)/(1-\mathrm{Le}_0)+\mathrm{Le}_0 B_2 k_4]\Big[(1+\sqrt{H_1})M\lambda_l/\lambda_0+$$
$$+B_2\Big(M-1+\frac{1}{\Gamma}\Big)\Big]\Big/\Big[k_3(1-k_0)\Big(1-\frac{B_2}{B_1}\Big)\Big/3(1-\mathrm{Le}_0)-A_4\Big]. \quad (1.19)$$

In the special case of a vapor bubble (k_0 = 1) the transcendental equation (1.19) coincides with the analogous equation obtained in [6].

2. Asymptotic Solutions of the Characteristic Equation

In the case of sufficiently large bubbles, when the influence of heat and mass transfer on their dynamics is slight, we can neglect capillary effects and seek a solution of Eq. (1.19) in the form

$$H=\sqrt{3\Gamma}\Pi(\cos\varphi+i\sin\varphi),\ \sin\varphi\approx 1,\ \cos\varphi<0,\ |\cos\varphi|\ll 1. \quad (2.1)$$

The following relations hold over a sufficiently broad range of the parameters of the system liquid + vapor–gas bubble:

$$\frac{\lambda_l}{\lambda_0}\gg 1,\quad \frac{a_l}{a_0}\ll 1,\quad \mathrm{Le}\sim 1,\quad k_1\sim 1,$$
$$|B_2|\ll\frac{\lambda_l}{\lambda_0}|1+\sqrt{H_1}|,\quad \Big|k_3(1-k_0)\frac{1-B_2/B_1}{1-\mathrm{Le}_0}\Big|\ll\frac{1}{\varkappa\,\mathrm{Le}_0}. \quad (2.2)$$

Using the representation (2.1) and the bounds (2.2), we can find an asymptotic solution of Eqs. (1.19) for $\Pi\gg 1$ [7]. We obtain an expression for the logarithmic decrement of the oscillations of large vapor–gas bubbles:

$$\Lambda_T=-2\pi\cos\varphi=3\pi\frac{\psi}{\sqrt{\mathrm{Pe}}},\quad \Pi\gg 1,\quad \sqrt{\mathrm{Pe}}\gg\psi,$$
$$\psi=\Gamma-1+k_0\left[\frac{k_0 B_0}{(\Gamma-1)B_V\varkappa^2}\frac{\lambda_0}{\lambda_l}\sqrt{\frac{a_l}{a_0}}+\frac{(1-k_0)B_g}{\Gamma B_0}\sqrt{\frac{a_0}{D}}\right]^{-1}, \quad (2.3)$$

where $\mathrm{Pe}=2R_0\sqrt{3\Gamma p_0/\rho_l}/a_0$ is the Péclet number.

In special cases (k_0 = 0 and D = 0), Eq. (2.3) coincides with the expression for the logarithmic decrement for the thermal damping of gas bubble oscillations without phase transitions [8], and for k_0 = 1 coincides with the corresponding expression for vapor bubbles [9, 10].

For a gas bubble and a vapor bubble Eq. (2.3) has the respective forms

$$\Lambda_T=\frac{3\pi(\Gamma-1)}{\sqrt{\mathrm{Pe}}}; \quad (2.4)$$

$$\Lambda_T=\frac{3\pi(\Gamma-1)}{\sqrt{\mathrm{Pe}}}\left[1+\varkappa^2\frac{\lambda_l}{\lambda_0}\sqrt{\frac{a_V}{a_l}}\right]. \quad (2.5)$$

Equation (2.3) can be rewritten

$$\Lambda_T=\frac{3\pi f(k_0)}{\sqrt{\Pi e}},\quad \Pi e=\frac{2R_0}{a_l|_{k=0}}\sqrt{\frac{p_0}{\rho_l|_{k=0}}},$$
$$f(k_0)=\psi\sqrt{\frac{a_0}{a_l|_{k=0}}}(3\Gamma\rho_l|_{k=0}/\rho_l)^{-1/4}. \quad (2.6)$$

This is a convenient representation insofar as Pe does not depend on k_0, and the dependence of Λ on k_0 is manifested only through the function $f(k_0)$.

3. Damping of Free Oscillations

The function $f(k_0)=\Lambda_T\sqrt{\mathrm{Pe}}/3\pi$ is plotted in Fig. 1 for a steam–air bubble in water at atmospheric pressure. The interdiffusion coefficient of the steam–air components of the mixture was varied. Curve 1 was calculated according to the actual value of D, which is given by the equation [11] $D=D_0(T/273)^n$, where $D_0=2.16\cdot 10^{-5}$ m²/sec and n = 1.8. Curves 2-5 correspond to D = 10^{-5}, 10^{-6}, 10^{-7}, and 10^{-8} m²/sec, respectively. We see that the function $f(k_0)$ is highly nonlinear. In the interval of k_0 close to unity, where the inequality $1-k_0\ll 1$ holds, we obtain Eq. (2.3) in the asymptotic form

$$\psi=(\Gamma-1)\left\{1+\frac{\lambda_l}{\lambda_0}\sqrt{\frac{a_0}{a_l}}\varkappa^2\left[1-(1-k_0)\frac{\lambda_l}{\lambda_0}\sqrt{\frac{a_0}{a_l}}\ (\Gamma-1)\varkappa^2\frac{B_g}{\Gamma B_V}\sqrt{\frac{a_0}{D}}\right]\right\}. \quad (3.1)$$

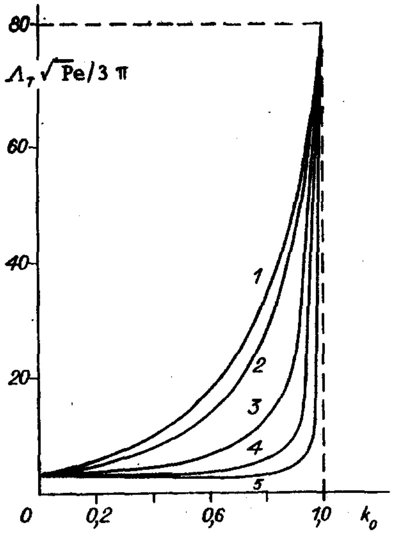

Fig. 1

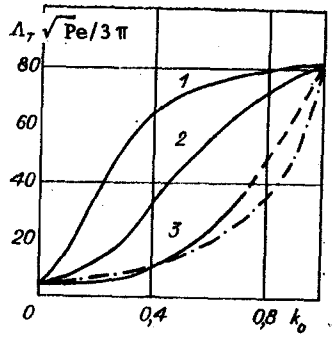

Fig. 2

It is evident from Fig. 1 and from Eq. (3.1) with regard for (2.2) that the addition of a minute quantity of inert gas to a vapor bubble reduces the damping of its oscillations significantly. This effect is particularly conspicuous for small values of the diffusion coefficient. This is attributable to the fact that the phase transition rate decreases with a decrease in D, since the vapor component loses its ability to penetrate rapidly through the shielding gas layer on the surface of the bubble.

It is also important to investigate the behavior of vapor bubbles for large values of the diffusion coefficient (formally, in the limit $D \to \infty$). This situation cannot be treated as a special case of Eq. (2.3), because its derivation was based on the representation $B_1 = \sqrt{H}$, which is valid for large values of $H = hR_0^2/D$. The following representation of B_1 is valid for sufficiently large values of D ($D = \infty$) and, hence, for small H:

$$B_1 = \sqrt{H}\,\mathrm{cth}\sqrt{H} - 1 = H/3, \quad |H| \ll 1. \tag{3.2}$$

Carrying out analogous calculations associated with the evaluation of the real part of the function Φ (1.19) and taking (3.2) into account, we obtain an expression for the logarithmic decrement of the vapor–gas bubble:

$$\Lambda_T = -2\pi\cos\varphi = 3\pi\left[\frac{\Gamma-1}{\sqrt{\mathrm{Pe}}} + \frac{\sqrt{\mathrm{Pe}}}{(\mathrm{Pe} + 2c_1\sqrt{\mathrm{Pe}} + c_1^2)}\right],$$
$$c_1 = 3\varkappa\frac{1-k_0}{k_0}\frac{MB_g}{B_0}\frac{\lambda_l}{\lambda_0}\sqrt{\frac{a_0}{a_l}}. \tag{3.3}$$

It is evident that the expression for the logarithmic decrement in the case $D = \infty$ has a more complicated form (in particular, a more complicated dependence on the bubble radius) than for the real value of D [Eq. (2.3)].

Figure 2 shows $\Lambda_T\sqrt{Pe}/3\pi$ as a function of the equilibrium vapor concentration k_0 for oscillations of a steam–gas bubble in water at atmospheric pressure. Curves 1-3 correspond to values of the bubble radius $R_0 = 10^{-2}$, 10^{-3}, and 10^{-4} m and were calculated according to Eq. (3.3). Curve 3 is shown dashed in the interval $k_0 \gtrsim 0.7$, because the oscillations of such small bubbles decay rapidly for large vapor contents, and the assumptions (2.1) underlying the solution no longer hold. The dot–dash curve corresponds to the real value of the diffusion coefficient (curve 1 in Fig. 1). We see that the simplifying assumption of the absence of diffusion resistance or a uniform concentration in the bubble for large bubbles creates an appreciable disparity with the solution obtained for real values of the diffusion coefficient. This simplification is valid for small vapor–gas bubbles ($R_0 \lesssim 10^{-4}$ m).

Figure 3 shows the logarithmic decrement of the oscillations of gas and vapor bubbles at atmospheric pressure as a function of the equilibrium radius R_0, calculated according to Eqs. (2.4) and (2.5). Curves 1-4 correspond to the following systems: 1) steam bubble in water at T = 373 K; 2) helium bubble in water at T = 300 K; 3) air bubble in water at T = 300 K; 4) vapor bubble in liquid helium at T = 4.2 K. It is evident from Fig. 3 that the damping of a steam bubble in water is much stronger than the damping of an air bubble, on account of phase transition. The damping of a bubble filled with helium gas is also much stronger than for an air bubble, owing to the large thermal diffusivity of helium. It is interesting that the damping of a vapor bubble in liquid helium at the boiling point for atmospheric pressure is far weaker, despite the occurrence of phase transitions (it is much weaker than the damping of a helium bubble without phase transitions; curve 2). This is attributable to the low values of the thermal conductivities of liquid helium and its vapor and especially to the high density of the vapor at low temperatures. In this case the density ratio of the phases $\rho_\ell/\rho_V \sim 10$, whereas for curves 1-3 $\rho_\ell/\rho_V \sim 10^3$ [11].

The damping component associated with the acoustic radiation of energy from the oscillating bubble in a compressible liquid must also be taken into account in the oscillations of large bubbles ($R \gtrsim 1$ mm). The expression for this component has the form [1]

$$\Lambda_a = \frac{\pi}{a}\sqrt{\frac{3\Gamma p_0}{\rho_l}} \tag{3.4}$$

(a is the sound velocity in the liquid). For small bubbles it is important to include the damping component associated with the viscosity of the liquid [1]:

$$\Lambda_\mu = \frac{4\pi\mu}{\rho_l R_0}\sqrt{\frac{\rho_l}{3\Gamma p_0}} \tag{3.5}$$

(μ is the viscosity coefficient of the liquid).

The total logarithmic decrement Λ can be determined for $\Lambda < 1$ as the sum of the individual components:

$$\Lambda = \Lambda_T + \Lambda_a + \Lambda_\mu\,. \tag{3.6}$$

The theoretical curves calculated according to Eqs. (2.4), (2.5), and (3.4)-(3.6) are compared in Fig. 4 with experimental data [12] on the damping of radial oscillations of large ($R \sim 10$ mm) steam (circles, 100°C) and air (light circle, 20°C) bubbles in water (curves 1 and 2, respectively).

Morioka [12], who published the first experimental data on the damping of oscillations of vapor and gas bubbles, but without any reference to the papers in which Eq. (2.5) was derived [9, 10], gave the dashed curve for steam bubbles, which was calculated according to Eq. (2.4) for the thermal damping of a gas bubble, except that the corresponding parameters of the vapor were used instead of the thermophysical properties of the gas (mainly the adiabatic exponent). Since the thermophysical properties of steam under standard conditions do not differ much from the properties of air, this curve is close to the corresponding curve for an air bubble in water. The disregard of phase transitions created a discrepancy of more than an order of magnitude with the experimental data on the damping of vapor bubble oscillations.

It is evident from Fig. 4 that the theoretical curves calculated according to Eq. (3.6) with the application of expressions (2.4), (2.5), and (3.4) exhibit good agreement with the experimental results both for air bubbles and for steam bubbles in water.

4. Oscillations of Bubbles in a Sound Field

In the case of small oscillations of bubbles under the action of a sound pressure $p_A \times \exp(i\omega t)$, the bubble radius can be described by the real part of the expression

$$R = R_0[1 + \alpha \exp(i\omega t)], \quad p_A \ll p_\infty, \tag{4.1}$$

where α is a complex number corresponding to the condition $|\alpha| \ll 1$, ω is the angular frequency of the sound field, and p_∞ is the hydrostatic pressure. As in the case of free oscillations, we assume that the small deviations of all the parameters from the equilibrium state can be represented in the form $A = A^0 \exp(i\omega t)$ (where A^0 is the amplitude of an arbitrary parameter).

Solving the system of linearized equations as in the case of free oscillations, we obtain the equation for the oscillation amplitude [3]

$$\alpha = p_A\left(\rho_l\omega^2 R_0^2 + \frac{2\sigma}{R_0} - 4i\omega\mu - \frac{3\Gamma p_0}{E}\right)^{-1},$$

$$E = 1 - 3\Gamma B_2(M - 1 + 1/\Gamma)Z_2^{-1} - \Gamma E_1 E_2,$$

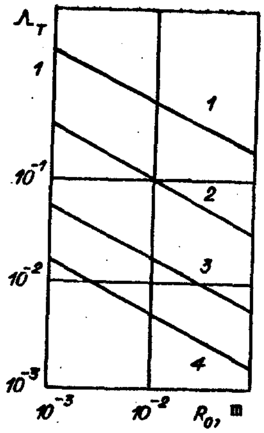

Fig. 3

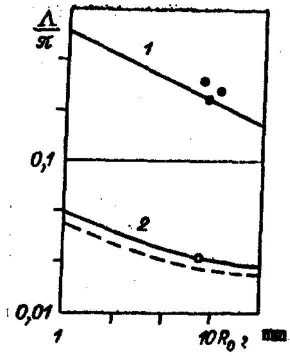

Fig. 4

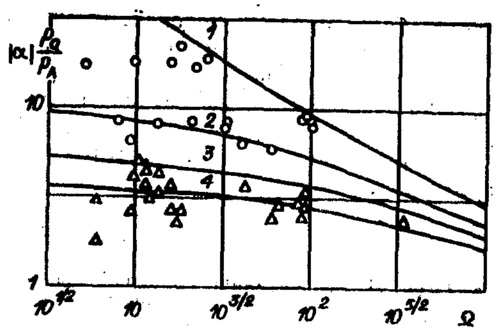

Fig. 5

$$E_1=\frac{1+k_2(1-k_0)}{Z}+\frac{3B_2k_4}{Z_2}+\frac{k_3(1-k_0)}{Z_2(1-\mathrm{Le}_0)}\left(\frac{B_2}{B_1}-1\right),$$

$$E_2=\frac{(1+\sqrt{i\Omega})\,M\lambda_l/\lambda_0+B_2(M-1+1/\Gamma)}{k_3(1-k_0)(1-B_2/B_1)/3(1-\mathrm{Le}_0)-B_2k_4-1/3\varkappa\,\mathrm{Le}_0-k_4(1+\sqrt{i\Omega})\,L},\tag{4.2}$$

$$\Omega=\frac{\omega R_0^2}{a_l},\quad Z=\frac{i\Omega a_l}{D},\quad Z_2=\frac{i\omega a_l}{a_0},$$

$$B_1=\sqrt{Z}\,\mathrm{cth}\sqrt{Z}-1,\quad B_2=\sqrt{Z_2}\,\mathrm{cth}\sqrt{Z_2}-1,\quad L=\frac{\lambda_l}{\lambda_0}.$$

The analytical relation (4.2) obtained for the response function of a vapor–air bubble in water as a function of the dimensionless frequency Ω is compared with the experimental data [13] in Fig. 5. The results of very delicate experiments to determine the compressibility of vapor–air bubbles executing radial oscillations in a sound field are given in [13] for small subcooling. The scatter of the bubble radii fell within the limits $0.02\ \mathrm{cm} < R_0 < 0.11\ \mathrm{cm}$. The temperature of the system at atmospheric pressure was varied in the interval $370\ \mathrm{K} < T < 373\ \mathrm{K}$, and the frequency of the sound field was varied in the interval $0.1\ \mathrm{Hz} < f = \omega/2\pi < 150\ \mathrm{Hz}$.

Assuming that the system liquid + vapor–gas bubble is in thermal and mechanical equilibrium in the absence of a sound field, we can relate the degree of subcooling of the system to the gas content of the bubble. Assuming that the partial pressure of the vapor component corresponds to the saturation condition, we obtain the following from the equations of state of a calorically ideal gas (1.1): $p_S(T_0)/(p_0-p_S(T_0))=k_0\mu_g/((1-k_0)\mu_V)$, where μ_g and μ_V are the molecular masses of the gas and vapor components. Accordingly $k_0=\left[1+\frac{\mu_g}{\mu_V}\left(\frac{p_0}{p_S(T_0)}-1\right)\right]^{-1}$. For a steam–air bubble in water at atmospheric pressure, the temperatures T = 372, 371, 370 K correspond to $k_g = 1 - k_0 = 0.05$, 0.1, 0.15. These values are represented by curves 2-4, respectively, in Fig. 5, and curve 1 was calculated for a steam bubble (T = 373 K); the experimental points are given for $372\ \mathrm{K} < T < 373\ \mathrm{K}$ (light circles) and for $370\ \mathrm{K} < T < 371\ \mathrm{K}$ (light triangles).

The expression (4.2) for the bubble compressibility is simplified in application to the experimental conditions reported in [13]:

$$\alpha\frac{p_\infty}{p_A}=-\frac{(1+\sqrt{i\Omega})}{i\Omega}M\frac{\lambda_l}{\lambda_0}\frac{a_0}{a_l}\left[\frac{1}{\varkappa}+\frac{(1-k_0)M\mu_V\,\mathrm{Le}_0\,\lambda_l}{k_0B_1\mu_g\lambda_V}(1+\sqrt{i\Omega})\right]^{-1}.\tag{4.3}$$

Equation (4.3) is further simplified for a vapor bubble:

$$\alpha\frac{p_\infty}{p_A}=-\frac{(1+\sqrt{i\Omega})\,\varkappa M}{i\Omega}\frac{\lambda_l a_V}{\lambda_V a_l}.$$

Equation (4.3) is simplified for the interval $10 < \Omega < 10^2$, in which the bulk of the experimental data was obtained in [13], because the function B_1 can be represented in the form $B_1=i\Omega a_l/3D$. The final expression for the response function now has the form

$$|\alpha|\frac{p_\infty}{p_A}=\frac{x}{\sqrt{\Omega(V^2+2Vy+2y^2)}},\quad x=\frac{M\lambda_l a_V}{3\lambda_V a_l},\quad V=\frac{1}{3\varkappa},\quad y=3x\frac{\mu_V(1-k_0)}{\mu_g\sqrt{2\Omega}}.$$

For $\Omega \lesssim 1$ the response function of the vapor–gas bubble obeys the equations (which were first published in [3])

$$\lim_{\Omega\to 0}|\alpha|\frac{p_\infty}{p_A}=\left[\frac{3(1+S_1)}{1+\mu_g k_0/\mu_V(1-k_0)}-S_1\right]^{-1},\quad S_1=\frac{2\sigma}{R_0 p_\infty}.$$

It is evident from Fig. 5 that the addition of a minute quantity of air (k_g = 0.05) to a vapor bubble produces a significant (almost an order of magnitude) decrease in the compressibility of the oscillating bubble at low frequencies. The calculated curves are in good agreement with the experimental data. Capillary effects were small ($S \ll 1$) for the experimental bubble sizes in [13]. Consequently, the dependence of the response function on the bubble radius is expressed only in terms of the dimensionless frequency $\Omega=\omega R_0^2 a_l^{-1}$.

Since the experiments reported in [13] were conducted for small bubbles at low sound frequencies, an analysis can also be carried out on the basis of simpler models without nonuniform temperature and concentration distributions in the pulsating bubble, consistent with the above-indicated simplifications of the general equation (4.2). However, situations are possible in which the failure to take these factors into account and the use of simplified models can induce appreciable errors.

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LITERATURE CITED

1. L. I. Sedov, A Course in Continuum Mechanics, Vol. 2, Wolters-Noordhoff, Groningen (1972).
2. R. I. Nigmatulin, Fundamentals of the Mechanics of Heterogeneous Media [in Russian], Nauka, Moscow (1978).
3. F. B. Nagiev and N. S. Khabeev, "Heat-transfer and phase-transition effects associated with oscillations of vapor–gas bubbles," Akust. Zh., 25, No. 2, 271 (1979).
4. M. Fanelli, A. Prosperetti, and M. Reali, "Radial oscillations of gas–vapor bubbles in liquids. Part 1: Mathematical formulation," Acustica, 47, 253 (1981).
5. M. Fanelli, A. Prosperetti, and M. Reali, "Radial oscillations of gas–vapor bubbles in liquids. Part 2: Numerical examples," Acustica, 49, 98 (1981).
6. N. S. Khabeev, "Heat-transfer and phase-transition effects in the oscillation of vapor bubbles," Akust. Zh., 21, No. 5, 815 (1975).
7. N. S. Khabeev and V. Sh. Shagapov, "Oscillations of a vapor–gas bubble in a sound field," Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 3 (1986).
8. C. Devin Jr., "Survey of thermal, radiation, and viscous damping of pulsating air bubbles in water," J. Acoust. Soc. Am., 31, No. 12 (1959).
9. R. I. Nigmatulin and N. S. Khabeev, "Decay rates of oscillations and effective heat-transfer coefficients of bubbles executing radial pulsations in a liquid," Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 6 (1980).
10. R. I. Nigmatulin, N. S. Khabeev, and F. B. Nagiev, "Dynamics, heat and mass transfer of vapor–gas bubbles in a liquid," Int. J. Heat Mass Transfer, 24, No. 6 (1981).
11. N. B. Vargaftik, Handbook of the Thermophysical Properties of Gases and Liquids [in Russian], Nauka, Moscow (1972).
12. M. Morioka, "Measurement of natural frequency and damping constant of single steam bubble oscillating in water," J. Nucl. Sci. Tech., 20, No. 10 (1983).
13. Y. Y. Hsu and R. G. Watts, "Behavior of a vapor bubble in a pulsating pressure field," in: Proc. Fourth Int. Conf. Heat Transfer, Vol. 5, Paris–Versailles (1970), Ser. B 2.4.